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MAY 77 M R LEADBETTER, G LINDGREN, H ROOTZEN N00014-75-C-0809

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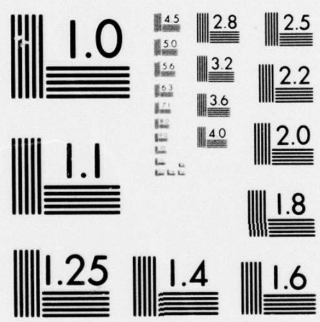
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CONDITIONS FOR THE CONVERGENCE IN DISTRIBUTION OF MAXIMA OF STATIONARY NORMAL PROCESSES*

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Conditions for the convergence in distribution of maxima of stationary normal processes.

Referat (sammandrag) The asymptotic distribution of the maximum $M_n = \max_{1 \leq t \leq n} \xi_t$ in a stationary normal sequence $\xi_1, \xi_2, \ldots$ depends on the correlation r_t between ξ_0 and ξ_t. It is well known that if $r_t \log t \to 0$ as $t \to \infty$ or if $\Sigma r_t^2 < \infty$, then the limiting distribution is the same as for a sequence of independent normal variables. Here it is shown that this also follows from a weaker condition, which only puts a restriction on the number of t-values for which $r_t \log t$ is large. The condition gives some insight into what is essential for this asymptotic behaviour of maxima. Similar results are obtained for a stationary normal process in continuous time.

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CONDITIONS FOR THE CONVERGENCE IN DISTRIBUTION
OF MAXIMA OF STATIONARY NORMAL PROCESSES

by

M.R. Leadbetter, G. Lindgren, and H. Rootzén

1. INTRODUCTION

Let $\{\xi_t\}_{t=-\infty}^{\infty}$ be a stationary normal sequence with zero means, unit variances and covariances $r_\tau = E(\xi_t \xi_{t+\tau})$, and put $M_n = \max_{1\le t\le n} \xi_t$. If $r_\tau = 0$, $\tau \neq 0$, i.e. if the variables are independent then

$$(1.1)\qquad P(a_n(M_n - b_n) \le x) \to e^{-e^{-x}}, \qquad n \to \infty,$$

where $a_n = \sqrt{2 \log n}$ and $b_n = a_n - \frac{1}{2} a_n\{\log \log n + \log 4\pi\}$. This result goes back to Fisher & Tippet (1928). The same conclusion was obtained under successively weaker dependence restrictions by Watson (1954), Loynes (1965), and Berman (1964). Berman's result is that if either (i) $r_n \log n \to 0$ as $n \to \infty$, or (ii) $\Sigma_{n=0}^{\infty} r_n^2 < \infty$ then (1.1) holds. Mittal & Ylvisaker (1975) considered a somewhat weaker version of (i) (in the vein of (2.2') below) and from their paper it can be seen that (i) is rather close to what is possible: if e.g. $r_n \log n \to \gamma > 0$ then a different limit law holds. Nevertheless neither of (i) and (ii) implies the other in general, and the precise relation between the conditions is not obvious.

For a standardized stationary normal process $\{\xi(t), -\infty<t<\infty\}$ in continuous time with covariance function $r(\tau) = E(\xi(t)\xi(t+\tau))$ the asymptotic behaviour of $M(T) = \max_{0\le t\le T} \xi(t)$ depends not only on the rate of decay of $r(\tau)$ as $\tau \to \infty$, but also on the local behaviour of $r(\tau)$. If

$$(1.2)\qquad r(\tau) = 1 - C|\tau|^{\alpha} + o(|\tau|^{\alpha}), \qquad \tau \to 0,$$

where C is a constant (or, more generally, a function of slow growth) and $0 < \alpha \le 2$ then there is a version of $\xi(t)$ which has continuous sample paths, and if $r(\tau)$ decreases quickly enough, then for this version

$$(1.3)\qquad P(a_T(M(T)-b_T) \le x) \to e^{-e^{-x}}, \qquad T \to \infty,$$

where $a_T = \sqrt{2 \log T}$ and $b_T = a_T + a_T^{-1}\{(\frac{1}{\alpha} - \frac{1}{2})\log \log T +$ $+ \log[(2\pi)^{-1/2}C^{-1/2}H_\alpha 2^{(2-\alpha)/2\alpha}]\}$. This has been proved under various conditions by Rozanov & Volkonski (1959), Cramér (1965), and Berman (1971a) for $\alpha = 2$ and by Pickands (1969) and Berman (1971b) for $0 < \alpha \leq 2$. Pickands and Berman assumed in addition to (1.2) either of the two conditions (i') $r(t) \log t \to 0$ as $t \to \infty$, or (ii') $\int r(t)^2 \, dt < \infty$ (or $\int r(t)^p \, dt < \infty$, some $p > 0$). Again, neither one of (i') and (ii') implies the other.

In the present note we consider conditions which are weaker than (i) and (ii) (or (i') and (ii')) but which still imply that (1.1) or (1.3) holds. These conditions seem to contain more of what is essential for (1.1) and (1.3) and will also clarify the relation between (i) and (ii) and between (i') and (ii'). We treat the discrete time case in Section 2 and the continuous time case in Section 3.

2. DISCRETE TIME

In this section we shall show that the condition

$$(2.1) \qquad n^{-1} \sum_{k=1}^{n} |r_k| \log k \, e^{\gamma |r_k| \log k} \to 0, \text{ as } n \to \infty,$$

for some $\gamma > 2$, together with $r_n \to 0$ is sufficient for (1.1) to hold. Essentially Condition (2.1) prevents $r_n \log n$ from being too large too often.

Define $\theta_n(x) = \{k; 1 \leq k \leq n, |r_k| \log k > x\}$ and let $\nu_n(x)$ be the number of elements in $\theta_n(x)$. The content of Condition (2.1) can be further elucidated by considering the following slightly stronger condition

$$(2.2) \qquad n^{-1} \sum_{k=1}^{n} |r_k| \log k \to 0, \text{ as } n \to \infty, \text{ and}$$

$$\nu_n(K) = O(n^\eta) \quad \text{for some } K > 0, \eta < 1,$$

and the equivalent condition

(2.2') $\qquad \nu_n(\varepsilon) = o(n), \ \forall\, \varepsilon > 0$, and

$$\nu_n(K) = O(n^{\eta}) \quad \text{for some} \quad K > 0,\ \eta < 1.$$

Obviously (i) implies (2.2'). Further, if $\Sigma_{k=1}^{\infty}|r_k|^p < \infty$ for some $p > 0$ then, since $\Sigma_{k=1}^{\infty}|r_k|^p \geq \Sigma_{\theta_n(x)}|r_k|^p \geq \nu_n(x)(x/\log n)^p$, it follows that $\nu_n(x) = O((\log n)^p)$. In particular, taking $p = 2$ we see that also (ii) implies (2.2'), so that both (i) and (ii) are stronger than (2.2) and (2.2'). The following lemma states that (2.2) or (2.2') imply (2.1) and consequently that both (i) and (ii) imply (2.1).

LEMMA 2.1 If $r_n \to 0$ as $n \to \infty$, then (2.2), and (2.2') both imply (2.1).

PROOF It is easily seen that (2.2) and (2.2') are equivalent so we need only show that (2.2) implies (2.1). We have

$$n^{-1}\sum_{k=1}^{n}|r_k|\log k\, e^{\gamma|r_k|\log k} = n^{-1}\sum_{\substack{1\leq k\leq n\\ k\notin\theta_n(K)}}|r_k|\log k\, e^{\gamma|r_k|\log k} + \tag{2.3}$$

$$+\, n^{-1}\sum_{k\in\theta_n(K)}|r_k|\log k\, e^{\gamma|r_k|\log k},$$

and proceed to estimate the sums in the right member separately, assuming that (2.2) holds. Now

$$n^{-1}\sum_{\substack{k=1\\ k\notin\theta_n(K)}}^{n}|r_k|\log k\, e^{\gamma|r_k|\log k} \leq e^{\gamma K}\frac{1}{n}\sum_{k=1}^{n}|r_k|\log k \to 0, \qquad n \to \infty,$$

by the first part of (2.2). Since we assume that $r_n \to 0$, there is an integer N such that $\gamma|r_k| < (1-\eta)/2$ for $k \geq N$. Hence

$$n^{-1}\sum_{\substack{k\in\theta_n(K)\\ k\geq N}}|r_k|\log k\, e^{\gamma|r_k|\log k} \leq n^{-1}\nu_n(K)\log n\; n^{(1-\eta)/2},$$

which tends to zero as $n \to \infty$, by the second part of (2.2). As N is fixed, $n^{-1}\Sigma_{k=1}^{N}|r_k|\log k\exp(\gamma|r_k|\log k) \to 0$, and it follows that also the second term of the right hand side of (2.3) tends to zero, and thus that (2.1) is satisfied. $\square$

Even if (2.1) is weaker than (2.2) this is only by a slight margin. In fact, $n^{-1}\Sigma_{k=1}^{n}|r_k|\log k \leq n^{-1}\Sigma_{k=1}^{n}|r_k|\log k \exp(\gamma|r_k|\log k)$, so if (2.1) holds then $n^{-1}\Sigma_{k=1}^{n}|r_k| \log k \to 0$ which in turn implies that $\nu_n(\varepsilon) = o(n)$, $\forall\varepsilon > 0$.

THEOREM 2.2 If $r_n \to 0$ as $n \to \infty$ and (2.1) is satisfied then (1.1) holds, i.e. the distribution of the (normalized) maximum converges to the double exponential distribution.

As is shown in Berman (1964) we only have to prove the following lemma to obtain the theorem. We use the notation of Leadbetter (1974).

LEMMA 2.3 Suppose that r_n satisfies the hypothesis of Theorem 2.2, and let $u_n = x/a_n + b_n$. Then

$$n \sum_{k=1}^{n} |r_k| e^{-u_n^2/(1+|r_k|)} \to 0 \quad \text{as} \quad n \to \infty. \tag{2.4}$$

PROOF We only indicate the changes which have to be made in [5] p. 22 (or in [1] p. 510). As is shown there

$$e^{-u_n^2/2} \sim Ku_n/n, \qquad (n \to \infty) \tag{2.5}$$

$$u_n \sim (2 \log n)^{1/2}, \qquad (n \to \infty)$$

($a \sim b$ means $a = b(1+o(1))$) where K is a constant, whose value below may change from line to line. Further $\delta = \sup_{n\geq 1}|r_n| < 1$. Put $\beta = 2/\gamma$ and let α be a constant such that $0 < \alpha < \min(\beta, \frac{1-\delta}{1+\delta})$.

Split the sum in (2.4) into three parts, the first for $1 \leq j \leq [n^\alpha]$, the second for $[n^\alpha] < j \leq [n^\beta]$ and the third for $[n^\beta] < j \leq n$. In [5] it is shown that the first sum tends to zero.

Next, define $\delta_n = \sup_{m\geq n}|r_m|$ and note that $\delta_n \to 0$ as $n \to \infty$. Now writing $p = [n^\alpha]$ and $q = [n^\beta]$ we have for the second part of (2.4)

$$n \sum_{k=p+1}^{q} |r_k| e^{-u_n^2/(1+|r_k|)} \leq n^{1+\beta} e^{-u_n^2} e^{u_n^2\delta_p} \leq K\, n^{\beta-1} u_n^2\, e^{\delta_p u_n^2}$$

$$\leq K\, n^{\beta-1} u_n^2\, n^{2\delta_p},$$

which tends to zero by (2.5).

Finally, for the last part of (2.4) we have, using $e^{-u_n^2/2} \sim u_n/n \sim$ $\sim (2 \log n)^{1/2}/n$,

$$n \sum_{k=q+1}^{n} |r_k| e^{-u_n^2/(1+|r_k|)} \leq Kn \sum_{k=q+1}^{n} |r_k| (u_n/n)^{2/(1+|r_k|)} \leq$$

$$\leq Kn^{-1} \log n \sum_{k=q+1}^{n} |r_k| e^{2|r_k| \log n}$$

For $k > q$ we have $\log k \geq \beta \log n$, and hence this is not larger than

$$Kn^{-1} \sum_{k=q+1}^{n} |r_k| \log k \, e^{2/\beta |r_k| \log k} \leq Kn^{-1} \sum_{k=1}^{n} |r_k| \log k \, e^{\gamma |r_k| \log k},$$

where we have used $2/\beta = \gamma$. By (2.1) this tends to zero as $n \to \infty$, which concludes the proof of (2.4). □

3. CONTINUOUS TIME

For a process with continuous time, the constant α in the local covariance condition (1.2) influences the normalization needed to obtain the limit law (1.3) for the maximum. In fact, the value of α also affects the extent with which the maximum of $\xi(t)$ over an interval can be approximated by the maximum over a discrete set of points. Let $h(t)$ be any function and define

$$(3.1) \quad \begin{aligned} &\theta_T(h) = \{t;\ 0 < t \leq T,\ |r(t)| \log t > h(t)\} \\ &\ell_T(h) = \lambda(\theta_T(h)) = \text{Lebesgue measure of } \theta_T(h). \end{aligned}$$

In analogy with the conditions for discrete time we will place restrictions on the amount of time that $|r(t)| \log t$ is large by requiring that there is some function h with $h(t) \downarrow 0$ as $t \uparrow \infty$ such that

(3.2') $\quad \ell_T(h) = O(T/(\log T)^\gamma)$, for some $\gamma > \max(0, 1 - 1/\alpha)$

and some constant $K > 0$ such that

(3.2") $\quad \ell_T(K) = O(T^\eta)$, for some $\eta < 1$.

Obviously (i'), i.e. $r(t) \log t \to 0$ as $t \to \infty$, implies that $\theta_T(h)$ is uniformly bounded in T, for example with $h(t) = 2 \sup_{s \geq t} |r(s)| \log s$, so that (i') implies (3.2). Further, since $\int_0^T |r(t)|^p dt \geq \ell_T(h)(h(T)/\log T)^p$, (ii'), i.e. $\int_0^\infty r(t)^2 dt < \infty$, implies that $\ell_T(h) = O((\log T/h(T))^2)$ for all h, so that also (ii') implies (3.2).

<u>THEOREM 3.1</u> If $r(t) \to 0$ as $t \to \infty$ and (3.2) is satisfied, then (1.3) holds, i.e. the distribution of the (normalized) maxima converges to the double exponential distribution.

Following the routine in Berman (1971) and Leadbetter (1974) we need only prove the following lemma.

<u>LEMMA 3.2</u> If $r(t)$ satisfies the hypothesis of Theorem 3.1, if $u = u_T = x/a_T + b_T$, $q = g(T)/(\log T)^{1/\alpha}$, where $g(T) \to 0$ as $T \to \infty$, and if the convergence of $g(T)$ to zero is slow enough, then

$$(3.3) \qquad \frac{T}{q} \sum_{\varepsilon \leq kq \leq T} |r(kq)| e^{-u^2/(1+|r(kq)|)} \to 0$$

as $T \to \infty$.

<u>PROOF</u> Let $\delta(t) = \sup_{s \geq t} |r(s)|$, let β satisfy $0 < \beta < (1-\delta(\varepsilon))/(1+\delta(\varepsilon))$, and split the sum in (3.3) into two parts at $kq \approx T^\beta$, i.e. let Σ' be the sum over $\varepsilon \leq kq \leq T^\beta$ and Σ'' the sum over $T^\beta < kq \leq T$. Since

$$e^{-u^2/2} = O(1)/T$$

we can estimate Σ' simply from the number of terms,

$$\frac{T}{q} \Sigma' = \frac{T}{q} \sum_{\varepsilon \leq kq \leq T^\beta} |r(kq)| e^{-u^2/(1+|r(kq)|)}$$

$$\leq \frac{T}{q} \cdot \frac{T^\beta}{q} \cdot e^{-u^2/(1+\delta(\varepsilon))} \leq \frac{KT^{1+\beta-2/(1+\delta(\varepsilon))}}{q^2} \to 0$$

by the choice of β and q.

For the remaining sum Σ'' we need a bound on the number of terms for which $|r(kq)| \log kq$ is not bounded by a small function. Define, for any function h,

$$n_T(h) = \#\{k;\ T^\beta < kq \le T,\ |r(kq)|\log kq > h(kq)\}$$

in analogy with $\ell_T(h)$ in (3.1). Since $r(t)$ satisfies a Lipschitz condition at 0 it does so uniformly for all t. In fact, if $\alpha' < \min(1, \alpha)$ then

$$|r(t+h) - r(t)| \le C|h|^{\alpha'},$$

see Boas (1967), Theorem 1. We will use this to give a bound for $n_T(h)$ in terms of $\ell_T(h/2)$. Let γ be as in condition (3.2) and take α' such that $\alpha/(1+\gamma\alpha) < \alpha' < \min(1, \alpha)$. Note that we can always find such an α' and that $\frac{1}{\alpha'} - \frac{1}{\alpha} - \gamma < 0$. We will show that for all non-increasing functions h,

$$(3.4) \qquad n_T(h) \le C'(\log T/h(T))^{1/\alpha'}\ \ell_T(h/2),$$

if T is large enough. Since, for $t \ge kq$, $|r(t)|\log t \ge (|r(kq)| - C|t-kq|^{\alpha'})\log kq$ we see that if

$$|r(kq)|\log kq > h(kq)$$

and t is such that

$$kq < t < kq + \left(\frac{h(T)}{2C\log T}\right)^{1/\alpha'}$$

then

$$|r(t)|\log t > h(t)/2.$$

We have $q = g(T)/(\log T)^{1/\alpha}$ and thus $(h(T)/\log T)^{1/\alpha'}/q = \frac{h(T)^{1/\alpha'}}{g(T)}(\log T)^{-1/\alpha'+1/\alpha}$ where $\alpha > \alpha'$. Since we have a free choice in letting $g(T) \to 0$ as slowly as necessary, we may thus assume that $(h(T)/\log T)^{1/\alpha'} q \to 0$ as $T \to \infty$. This implies that for T large enough the kq which contribute to $n_T(h)$ also contribute disjoint intervals of length $(h(T)/(2C\log T))^{1/\alpha'}$ to $\ell_T(h/2)$, and we get (3.4) with $C' = (1/2C)^{1/\alpha'}$.

We can now proceed by splitting the sum Σ'' according to if $kq \in \theta_T(2K)$ or not. Recalling the notation $\delta(t) = \sup_{s \ge t}|r(s)|$, we have

$$\frac{T}{q}\Sigma'' = \frac{T}{q}\sum_{T^\beta<kq\leq T}|r(kq)|e^{-u^2/(1+|r(kq)|)} \leq$$

$$(3.5)\qquad \leq \frac{T}{q}n_T(2K)e^{-u^2/(1+\delta(T^\beta))} +$$

$$+\frac{T}{q}\sum_{T^\beta<kq\leq T,\ kq\in\theta_T(2K)^*}|r(kq)|e^{-u^2(1-2K/\log T^\beta)}$$

The first term in (3.5) is bounded by

$$\frac{T}{q}C'(\log T/2K)^{1/\alpha'}\ell_T(K)\ O(1)T^{-2/(1+\delta(T^\beta))} \leq$$

$$\leq C''\frac{(\log T)^{1/\alpha'+1/\alpha}}{g(T)}T^{1+\eta-2/(1+\delta(T^\beta))}.$$

Since $\eta < 1$ by (3.2) and $\delta(T^\beta) \to 0$ this bound goes to zero as $T \to \infty$ if $g(T) \to 0$ slowly enough.

The second term in (3.5) is bounded by

$$(3.6)\qquad (\frac{T}{q})^2 e^{-u^2(1-2K/(\beta\log T))}\frac{1}{\beta\log T}\cdot\frac{q}{T}\Sigma|r(kq)|\log kq = F_1\cdot F_2,$$

say, where the sum is extended over all kq such that $T^\beta < kq \leq T$ and $kq \in \theta_T(2K)^*$. We will see that $F_1 \to \infty$, $F_2 \to 0$ as $T \to \infty$, but that $F_1\cdot F_2 \to 0$. We start with F_2, introducing the function h that appears in (3.2') and split the sum according to wether $kq \in \theta_T(2h)$ or not, giving

$$F_2 = \frac{q}{T}\Sigma|r(kq)|\log kq \leq \frac{q}{T}\sum_{\substack{kq\in\theta_T(2h)^*\\ kq\leq T}} + \frac{q}{T}\sum_{kq\in\theta_T(2h)\cap\theta_T(2K)^*} \leq$$

$$\leq \frac{q}{T}\cdot\frac{T}{q}2h(T^\beta) + \frac{q}{T}\cdot 2Kn_T(2h) \leq 2h(T^\beta) + 2KC'\frac{q}{T}(\log T/h(T))^{1/\alpha'}\ell_T(h) =$$

$$= 2h(T^\beta) + \frac{g(T)}{(h(T))^{1/\alpha'}}(\log T)^{1/\alpha'-1/\alpha-\gamma}\cdot O(1) = 2h(T^\beta)+g(T)k(T),$$

say, by Condition (3.2') and the definition of g. Since $1/\alpha'-1/\alpha-\gamma < 0$, we can deduce that $k(T) \to 0$ as $T \to \infty$, provided $h(t)$ decreases sufficiently slowly. Also note that if (3.2') is fulfilled for some function h, then it is fulfilled for all functions which decrease more slowly. We can therefore assume that $k(T) \to 0$ as $T \to \infty$. The remaining factor F_1 in (3.6) is given by

$$F_1 = \left(\frac{T}{q}\right)^2 e^{-u^2(1-2K/(\beta \log T))} \frac{1}{\beta \log T} .$$

Using the fact that

$$u^2 = 2 \log T + 2\left(\frac{1}{\alpha} - \frac{1}{2}\right)\log \log T + O(1)$$

we get

$$F_1 = \frac{O(1)}{q^2 \log T} e^{-2(1/\alpha-1/2)\log \log T} =$$

$$= \frac{O(1)}{g(T)^2} (\log T)^{2/\alpha-1-2(1/\alpha-1/2)} = \frac{O(1)}{g(T)^2} .$$

Thus

$$F_1 \cdot F_2 \leq O(1)\left\{\frac{2h(T^\beta)}{g(T)^2} + \frac{k(T)}{g(T)}\right\},$$

where $k(T)$ does not depend on $g(T)$. Since we may let $g(T) \to 0$ arbitrarily slowly we obtain that $h(T^\beta)/g(T)^2 \to 0$ and $k(T)/g(T)^2 \to 0$ as $T \to \infty$, which completes the proof of the lemma. □

<u>REMARK 3.3</u> As in discrete time one would be inclined to consider a condition like

$$(3.7) \qquad \frac{q}{T} \sum_{T^\beta \leq kq \leq T} |r(kq)| \log kq \; e^{\gamma |r(kq)| \log kq} \to 0$$

as $T \to \infty$, for some $\beta < 1$, $\gamma > 2$. We can presently prove that (3.7) can replace (3.2) at least if $\alpha = 2$. However, (3.7) contains the somewhat arbitrary spacing q. A more natural condition for a continuous time process would restrict the size of

$$\int_1^T |r(t)| \log t \; e^{\gamma |r(t)| \log t} \, dt.$$

How this should be done in relation to (3.7) is not clear.

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It is a pleasure to thank Yashaswini Mittal for her help.

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In this report the standard ("Berman") conditions for the validity of the asymptotic distribution for the maximum of a normal process are further weakened, to become even closer to necessity. Both discrete and continuous parameter cases are considered.

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